# Design of Gratings and Frequency Selective Surfaces Using Fuzzy ARTMAP Neural Networks

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**ABSTRACT**

This paper presents a study of the Fuzzy ARTMAP neural network in designing cascaded gratings and Frequency Selective Surfaces (FSS) in general. Conventionally, trial and error procedures are used until an FSS matches the design criteria. One way of avoiding this laborious and manual process is to use neural networks. A neural network can be trained to predict the dimensions of the metallic patches(or apertures), their distance of separation, their shape, and the number of layers required in a multilayer structure which gives the desired frequency response. In the past, to achieve this goal, the back propagation (back-prop) learning algorithm was used in conjunction with an inversion algorithm. Unfortunalety, the back-prop algorithm sometimes has problems with convergence. In this work the Fuzzy ARTMAP neural network is utilized. The Fuzzy ARTMAP is faster to train than the back-prop and it does not require an inversion algorithm to solve the FSS problem. Most importantly, its convergence is guaranteed. Several results (frequency responses) from cascaded gratings for various angles of wave incidence, layer separation, width strips, and interstrip separation are presented and discussed.

## 1 Introduction

Frequency selective surfaces (FSS) have numerous applications as electromagnetic system devices such as, polarizers, filters, radomes, dichroic reflectors, infrared sensors and beam tuners for optical systems. Currently, there is no closed form solution that can directly relate a desired frequency response to the corresponding FSS. Trial and error procedures are used until a frequency selective surface matches the design criteria. One way of avoiding this laborious process and obtain a synthesis procedure is to utilize the training capabilities of neural networks. A neural network can be trained to predict the dimensions of the metallic patches(or apertures), their distance of separation, their shape, and the number of layers required in a multilayer structure, which gives the desired frequency response.

Previous work was limited to the designing of FFS using the back-prop method [1, 2] . During the training of the neural network, the geometric information (the dimentions, shapes etc.) is fed to the neural network as its input. The corresponding frequency response for each dimension and shape is also fed into the neural network as its desired output. Once the neural network has been trained, then for a given desired frequency response the network could determine the best dimensions (or range of dimensions) and shape that would yield such a frequency response.

In the back-prop method, once the desired frequency is fed to the trained neural network, an inversion algorithm is employed to yield the appropriate dimensions and element shapes that can generate such a response. The convergence and success of the network depend on the constraints applied to the inversion algorithm [3]. For that reason, the Fuzzy ARTMAP architecture is chosen to perform the same task. This architecture is faster to train than the back-prop and it does not require any added inversion algorithm. Most importantly, its convergence is guaranteed . Furthermore, once this network is implemented on hardware, one can continuously, keep on adding to the training of the Fuzzy ARTMAP network as new data are obtained. On the other hand the back-prop network has to be retrained every time some new training data are added.

In this paper, the Fuzzy ARTMAP is applied to the specific problem of cascaded gratings, with different strip widths, interstrip distance of separation, layer separation, and angles of wave incidence. The results obtained from this work along with the electromagnetic model and the Fuzzy ARTMAP architecture used, are presented and discussed.

## 2 Modeling of Cascaded Gratings

Gratings and dielectrics may be cascaded as shown in Figure 1. In general, for the $n^{th}$ grating the transmission matrix can be expressed as [4]:

$$TR_n = \begin{bmatrix} t_n(1 - r_n^2/t_n^2) & r_n e^{j2kL_n cos\theta_i}/t_n \\ -r_n e^{-j2kL_n cos\theta_i}/t_n & 1/t_n \end{bmatrix} \tag{1}$$

where:
$L_n = d_1 + d_2 + d_3 + \ldots + d_{n-1}$ for $n = 2, 3, \ldots N$
$t_n$ = transmission coefficient for that grating (dielectric) at $L_n$ distance.
$r_n$ = reflection coefficient for that grating(dielectric) at $L_n$ distance.
$N$ = total number of gratings and dielectric slabs
$k = 2\pi/\lambda$ is the wave number in free space
$\theta_i$ = incident angle between normal and incident vector $\vec{k}$.
t and r are calculated for any angle of incidence using the *Spectral Iteration Approach* whose brief description follows herein. Any techique can used instead to calculate the individual r's and t's.

Once the transfer matrix [TR] for each grating or dielectric slab is found the general form of the total cascaded structure is expressed as:

$$[TR_n][TR_{n-1}][TR_{n-2}]\ldots[TR_1] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{2}$$

$n = 1, 2, 3, \ldots N$

The total transmission and reflection coefficients are given by:
*Total Transmission*

$$T_t = A - (BC/D) \tag{3}$$

*Total Reflection*

$$R_t = -C/D \tag{4}$$

### 2.1 Spectral Iteration Approach

For a planar structure at z=0 plane, shown in Figure 2, it can be shown that the incident $\vec{H}^{inc}$ field can be expressed in terms of the aperture electric field $\vec{E}_a$ as :

$$\vec{H}^{inc} = -\frac{2}{j\omega\mu} \sum_{mn} \begin{bmatrix} \alpha_{mn}\,\beta_{mn} & k_o^2 - \alpha_{mn}^2 \\ \beta_{mn}^2 - k_o^2 & -\alpha_{mn}\,\beta_{mn} \end{bmatrix} \tilde{\bar{\bar{G}}}\,\tilde{\vec{E}}_a\; e^{j(\alpha_{mn}x + \beta_{mn}y)} \tag{5}$$

where the sign ( $\tilde{}$ ) denotes the Fourier transformed quantity. $\alpha_{mn}$ and $\beta_{mn}$ represent the Floquet coefficients which are defined as :

$$\begin{aligned} \alpha_{mn} &= 2\pi(m/a) - k_o\, sin\theta\, cos\phi \\ \beta_{mn} &= 2\pi(n/b\, sin\Omega) - 2\frac{m\pi}{a}\, cot\Omega + k_o\, sin\theta\, sin\phi \end{aligned} \tag{6}$$

and

$$\tilde{\bar{\bar{G}}}(\alpha_{mn}, \beta_{mn}) = (-j/2)\sqrt{k_o^2 - \alpha_{mn}^2 - \beta_{mn}^2}\;\bar{\bar{I}} \tag{7}$$

is the Fourier transform of Green's dyadic function.

The above equation applies only to the aperture region and in order to include the contribution of the $\vec{H}$ field along the conducting strips, the current densities have to be added to yield:

$$\overline{Tcr}(\vec{J}) = \hat{n} \times \vec{H}^{inc} + \frac{2}{j\omega\mu} \sum_{mn} \begin{bmatrix} \alpha_{mn}\,\beta_{mn} & k_o^2 - \alpha_{mn}^2 \\ \beta_{mn}^2 - k_o^2 & -\alpha_{mn}\,\beta_{mn} \end{bmatrix} \tilde{\tilde{G}}\,\vec{E}_a\; e^{j(\alpha_{mn}x+\beta_{mn}y)} \tag{8}$$

Because the current density can only be present on the strip, a truncation operator is used which is defined by:

$$Tcr\{X(\vec{r})\} = \begin{cases} X(\vec{r}) & \text{for } \vec{r} \text{ in the aperture} \\ 0 & \text{for } \vec{r} \text{ in the conducting region} \end{cases} \tag{9}$$

with $\overline{Tcr}[X(\vec{r})]$ = the opposite of Tcr

For gratings, $\Omega = 90^o$, and $b = \infty$. To solve for equation 8, an iterative scheme is used which is expressed in terms of the $\vec{E}$ field as [5]:

$$\vec{E}_t^{i+1} = F^{-1}\left[\Psi^{-1} F \left\{(j\omega\mu/2)\left[\overline{Tcr}\{\vec{H}_t^{inc} + (2/j\omega\mu)\,F^{-1}[\Psi\,F\,Tcr(\vec{E}_t^i)]\}\right] - \vec{H}_t^{inc}\right\}\right] \tag{10}$$

where :

$$\Psi = \begin{bmatrix} \alpha_{mn}\,\beta_{mn} & k_o^2 - \alpha_{mn}^2 \\ \beta_{mn}^2 - k_o^2 & -\alpha_{mn}\,\beta_{mn} \end{bmatrix} \tilde{\tilde{G}} \tag{11}$$

$F$ stands for the Fourier transform and $F^{-1}$ for its inverse.

# 3 Fuzzy ARTMAP

## 3.1 Fuzzy ARTMAP Architecture

Fuzzy ARTMAP [6] is a neural network architecture which can learn to approximate a piecewise-continuous function from $R^n$ to $R^m$. It consists of three modules: ARTa, ARTb and the Inter-ART module as shown in Figure 3. The ARTa and ARTb both have an architecture of Fuzzy ART [7] so that they can accept analog patterns as their inputs. Each of ARTa and ARTb classifies its input into an appropriate category by some similarity rule. The function of Inter-ART module is to learn the mapping between the pattern pair fed to ARTa and ARTb.

ARTa and ARTb each consists of three layers: $F_0$, $F_1$ and $F_2$. The preprocessing layer $F_0$ performs the complement-coding of its input, which is necessary for the successful operation of Fuzzy ART [7]. The $F_2$ layer is the category representation layer because its nodes denote the categories to which the inputs at the $F_0$ belong. The $F_1$ layer receives signals from both $F_0$ and $F_2$ and it evaluates whether an input pattern at the $F_0$ are close enough to the template of the chosen $F_2$ node. The criterion of the closeness is controlled by the vigilance parameter $\rho_a$ for ARTa and $\rho_b$ for ARTb. If a node $J$ in the $F_2$ layer is chosen and the closeness criterion is passed, then the learning of the connections associated with node $J$ ( i.e. the bottom-up weights $\mathbf{Z}_J$ and the top-down weights $\mathbf{z}_J$) occurs. Otherwise, a reset signal will be sent to $F_2$ layer and then a search for another node in $F_2$ starts. This procedure repeats until an appropriate node in the $F_2$ layer is found to represent the input pattern at the $F_0$ layer.

In order for Fuzzy ARTMAP to learn the mapping between an input pattern $\mathbf{I}_0$ and an output pattern $\mathbf{O}_0$, the input $\mathbf{I}_0$ should be fed to the $F_0$ layer of ARTa (i.e. ${F_0}^a$), and the output $\mathbf{O}_0$ to $F_0$ layer of ARTb (i.e. ${F_0}^b$). When an input-output pair is presented to Fuzzy ARTMAP, ARTa classifies the input $\mathbf{I}_0$ to an appropriate category represented by a node (e.g. $J$) in ${F_2}^a$, while ARTb classifies the output $\mathbf{O}_0$ to an appropriate category represented by a node (e.g. $K$) in the ${F_2}^b$. There are three cases concerning the learning in the Inter-ART: (1) No mapping between node $J$ in ${F_2}^a$ and any node in the ${F_2}^b$ has been established. Then the learning in the Inter-ART module occurs by setting $\mathbf{W}_{JK}$=1 and $\mathbf{W}_{jk}$=0 for any other j and k. (2) The mapping between node $J$ in ${F_2}^a$ and the node $K$ in ${F_2}^b$ has been established. In this case, no learning is necessary in the Inter-ART for this presented input-output pair. (3) The mapping between node $J$ in ${F_2}^a$ and a node in ${F_2}^b$ other than $K$ has been established. In this case, the vigilance parameter $\rho_a$ is increased by a minimum amount which causes node $J$ to reset, and then another node $\hat{J}$ is selected. Repeat this if $\hat{J}$ still falls in this case.

In order for Fuzzy ARTMAP to learn a list of input-output pairs, the pattern pairs should be repeatedly presented to the Fuzzy ARTMAP until all the input-output pairs are correctly mapped and no learning occurs.

### 3.2 Fuzzy ARTMAP applied to the design of cascaded gratings

During the training, ARTa is fed with the normalized grating parameters: incident angle $\theta$ (normalized by 100), physical sizes b and d (both normalized with a). ARTb is fed with the samples of the transmission coefficient curve corresponding to the grating parameters.

After the Fuzzy ARTMAP learned all the mapping of the training data, the desired set of transmission coefficients are fed to the $F_0$ layer of ARTb. Then a node in the ${F_2}^b$ (a category of ARTb) will be selected by the similarity rule. By the connections between this ${F_2}^b$ node, the nodes in the Inter-ART and the ${F_2}^a$ nodes, one or more corresponding ${F_2}^a$ node(s) will be picked. The weights associated with the picked ${F_2}^a$ node(s) determines the designed grating parameters. If the ${F_2}^a$ node has learned only one set of the normalized grating parameters, the designed grating parameters will be of single value. Otherwise, a range for either b or d or $\theta$ will be determined.

## 4 Results

Several cases of cascaded gratings were used to train the Fuzzy ARTMAP neural network. Basically, the width of the strips, their distance of separation, the distance of layer separation, and the angle of wave incidence were the parameters used for training of the neural network. For each change in the above parameters a different frequency response was obtained and fed to the network. Once the training was accomplished, the desired frequency responses were fed as inputs to the neural network which in return yielded the dimensions and parameters required to obtain such a response. Figures 4 through 10 are examples of results obtained from the ARTMAP neural network.

Figures 4 through 6 are cases where the desired response is very closed to one of the responses used to train the neural network. As one can see the Fuzzy ARTMAP can quickly and accurately match the desired frequency response. Figures 4 and 5 are for $\theta = 0^0$, whereas Figure 6 is for $\theta = 60^o$. Figure 7 depicts a case where the shape of the network response is similar to the desired one but it differs in magnitude. In this case, more data are required to cover such a response during the training of the neural network. In Figures 8 through 10, the network results in a number of responses that are close to the desired response within the limits determined by the closeness parameter ($\rho_b$). The smaller the papameter $\rho_b$ is, the more possibilities of responses that resemble the desired one will be given by the Fuzzy ARTMAP network. For example, in Figure 8, there are three responses that can be used to approximate the desired response. One corresponds to the parameters $\theta = 60^o$, d=1a, b=.3a, and the other two are for the same angle of wave incidence $\theta = 20^o$ and strip width but for different distances of layer separation. As it can be seen the one with $\theta = 20^o$ and d=0.5a is the best fit. Similarly, Figures 9 and 10 yield a range of solutions for various angles of wave incidence but one of them is always the best fit. To get more accurate design, $\rho_b$ should be set close to 1, and a larger training data set is needed. The tradeoff is that the Fuzzy ARTMAP will require more nodes in the $F_2$ layer in both $ART_a$ and $ART_b$, therefore more connections (weights).

## 5 Conclusions

The Fuzzy ARTMAP neural network is trained to synthesize a given desired frequency response. The electromagnetic model as well as the ARTMAP architecture used were presented with various results. Unlike the back-prop method, this network does not require any inversion algorithms to yield the dimensional parameters of the FSS. This method is capable of yielding a range of solutions, all close to the desired solution. This range depends on the closeness parameter chosen by the designer. This neural network architecture can be utilized in solving frequency selective surfaces with more complicated geometries.

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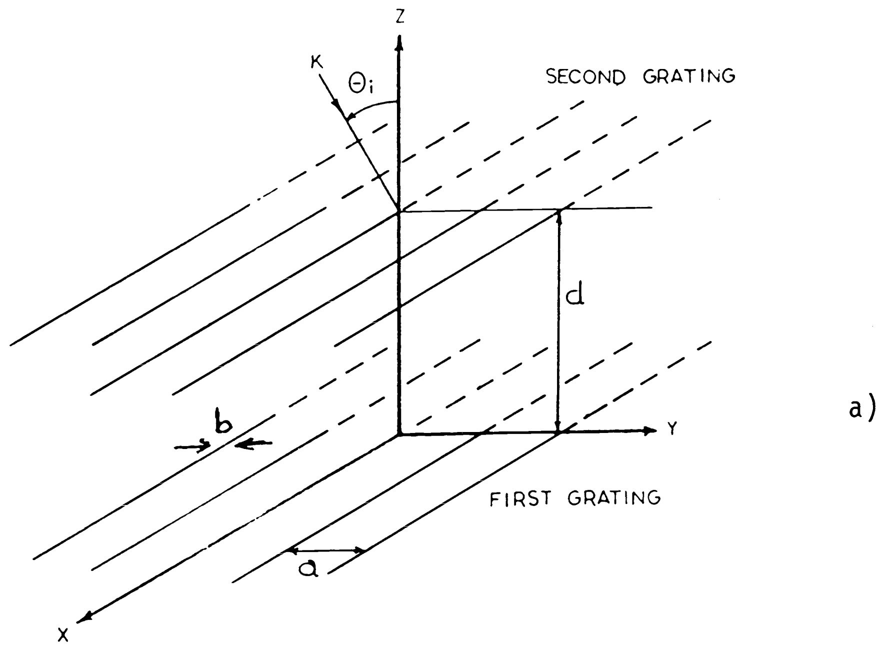


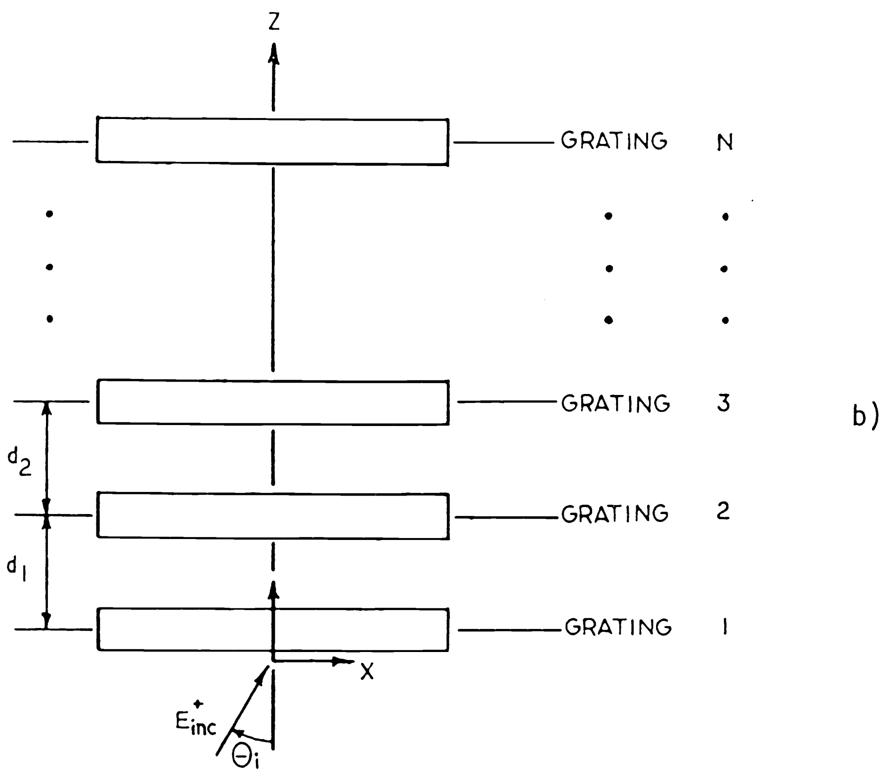


Figure 1: a) Geometry of two cascading gratings b) Cascading of N gratings or dielectric slabs

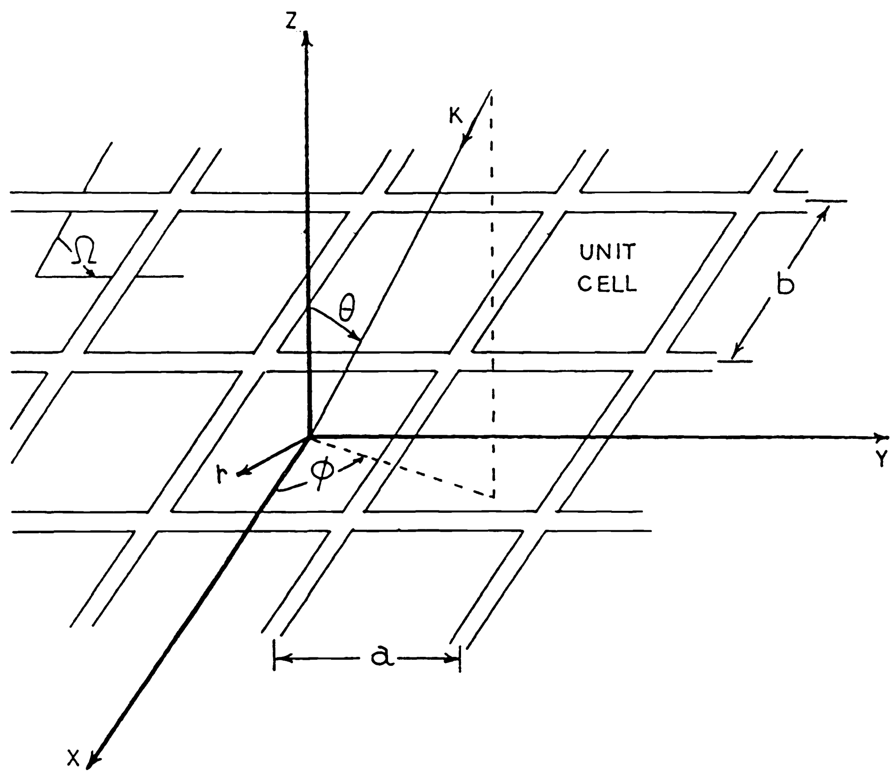


Figure 2: Geometry of a planar mesh structure

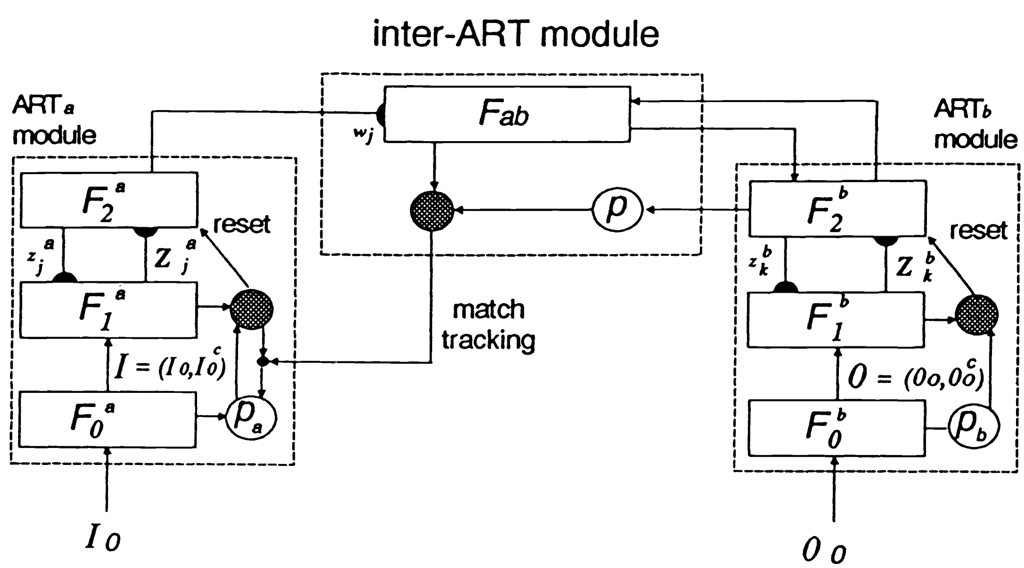


Figure 3: Fuzzy ARTMAP architecture

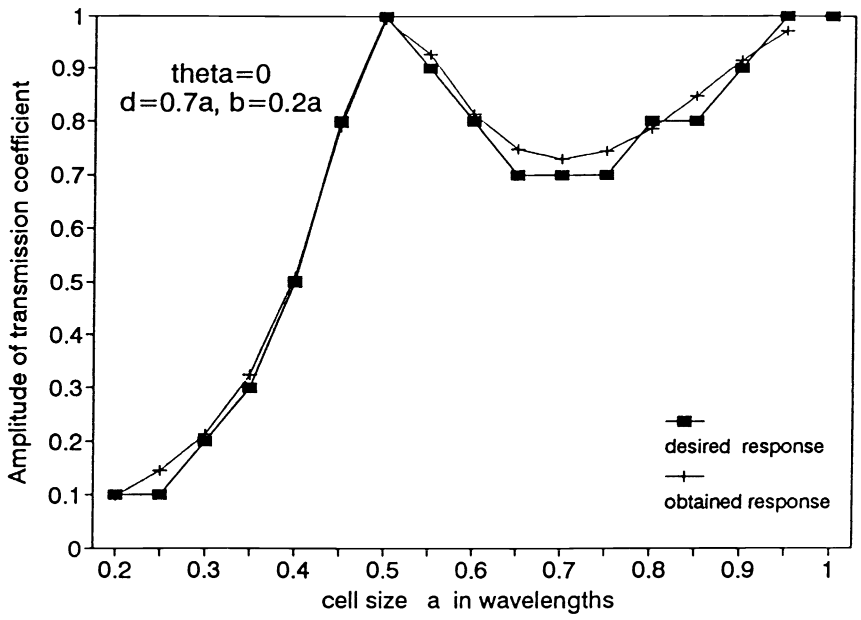


Figure 4: Amplitude of transmission coefficient for the desired and obtained response for two cascaded gratings. $\theta = 0^o$

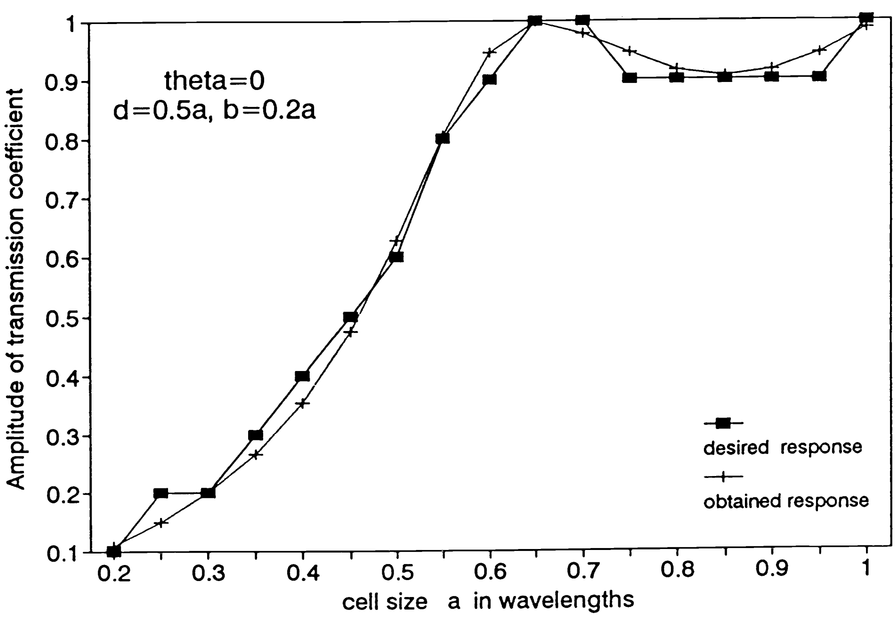


Figure 5: Amplitude of transmission coefficient for the desired and obtained response for two cascaded gratings. $\theta = 0^o$

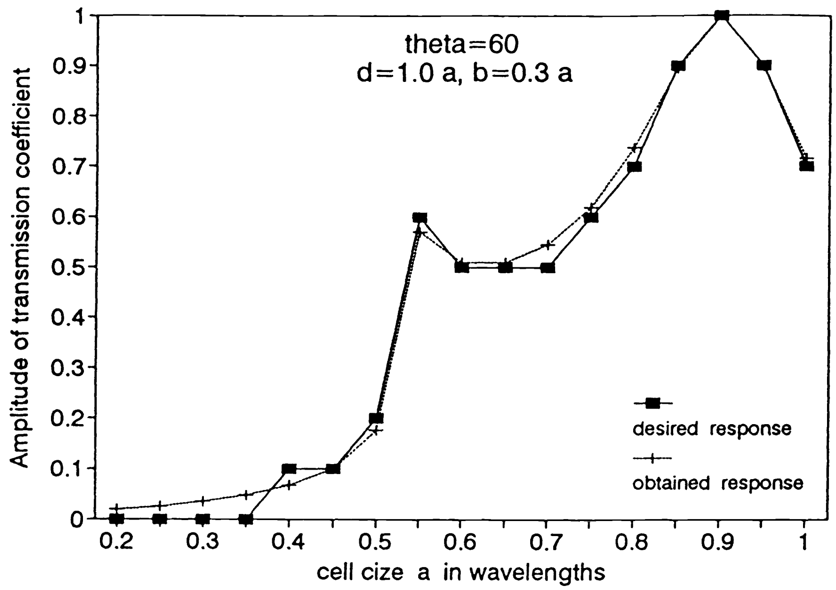


Figure 6: Amplitude of transmission coefficient for the desired and obtained response for two cascaded gratings. $\theta = 60^0$

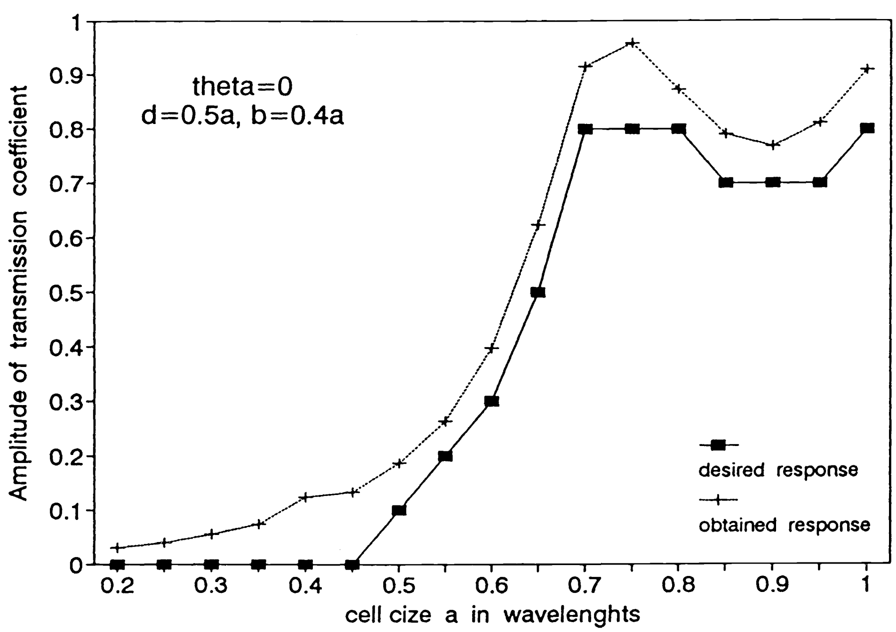


Figure 7: Amplitude of transmission coefficient for the desired and obtained response for two cascaded gratings. $\theta = 0°$. Desired response is chosen not to match, in magnitude, any of the data used to train the neural network

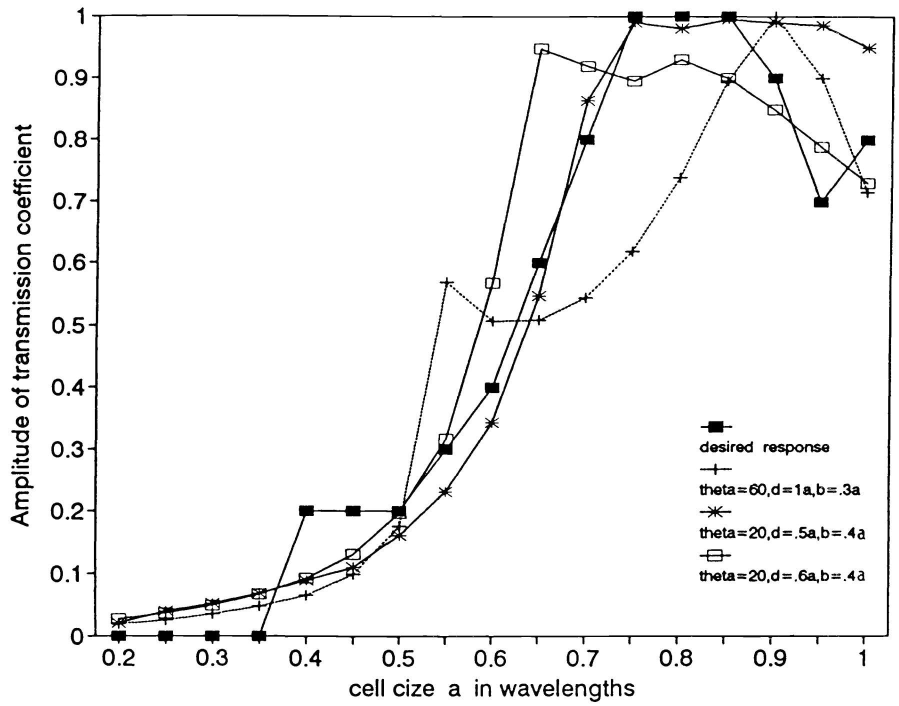


Figure 8: Amplitude of transmission coefficient for the desired and a range of obtained response for two cascaded gratings.

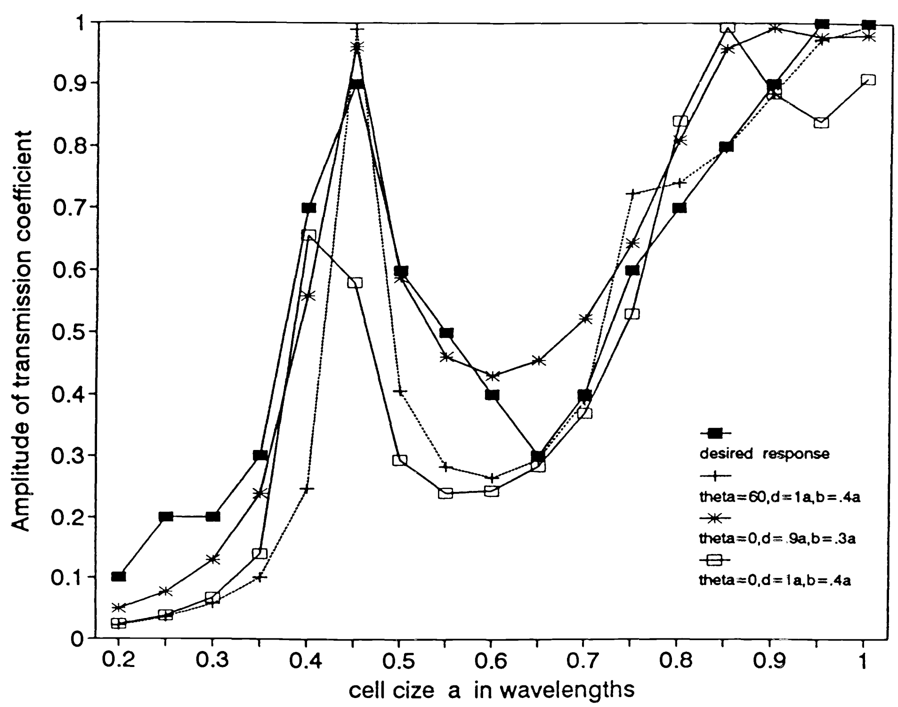


Figure 9: Amplitude of transmission coefficient for the desired and a range of obtained response for two cascaded gratings.

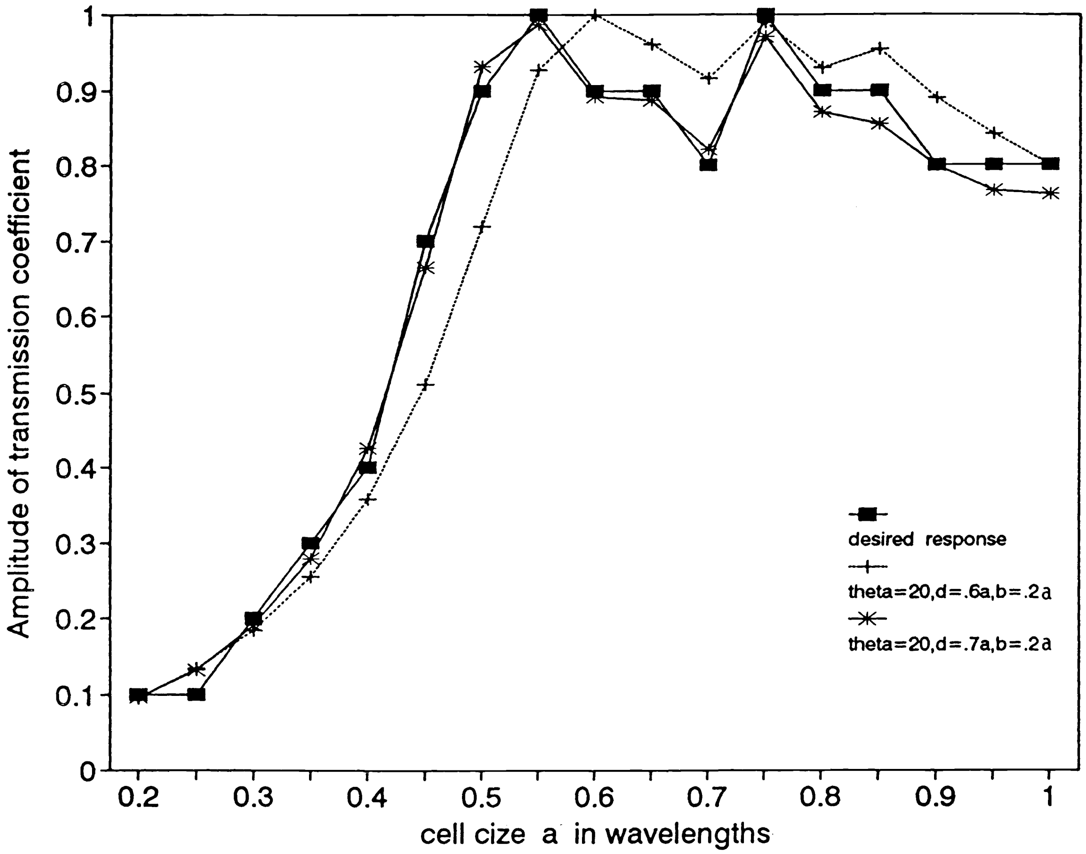


Figure 10: Amplitude of transmission coefficient for the desired and a range of obtained response for two cascaded gratings.